

USAR Roach 2

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Summary

The ROACH (Remotely Operated And Controlled Hexapod) robot is an ongoing design project used in the USAR (Urban Search And Rescue) annual competition. After analyzing the 2004-2005 design it was determined that to best utilize the newly purchased infrared camera it would be beneficial to raise the viewing level of the camera. After comparing multiple designs for elevating the camera, it was determined that the best design was a collapsible linkage system. This system provides maximum height extension, light weight, low cost, and low overall volume. One of the design considerations for this linkage system was the link itself. This paper presents the process and results of designing the link shape, choosing an appropriate material, and optimizing the weight of linkage using finite element analysis.

Introduction

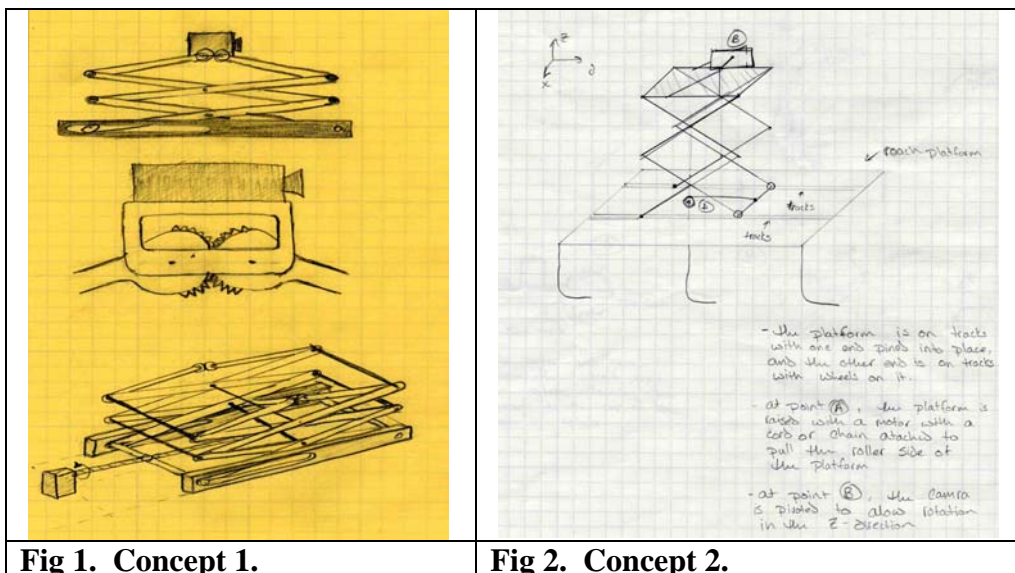
The RoboCup Rescue Robot competition, to be held in July, 2005, will host a senior design team from Colorado State University and their second year legacy project called ROACH, short for Remotely Operated And Controlled Hexapod. An explanation of the goal of the competition is given by the RoboCup organizers as follows:

Disaster rescue is (a) serious social issue which involves very large numbers of heterogeneous agents in the hostile environment. The intention of the RoboCup Rescue project is to promote research and development in this socially significant domain at various levels involving multi-agent team work coordination, physical robotic agents for search and rescue, information infrastructures, personal digital assistants, a standard simulator and decision support systems, evaluation benchmarks for rescue strategies and robotic systems that are all integrated into a comprehensive systems in future. [1]

Collapsed buildings, coal mines, avalanches, and other such 'hostile' environments necessitate the need for remotely controlled devices to locate survivors and map disaster areas without further risk to human life. Such is the impetus of the Rescue Robot competition.

Each year the competition incorporates new rules to test the capacity and reliability of the robots and design teams are constantly called upon to improve on existing designs as well as devise new concepts to be added to previous projects, hence the term 'legacy' project.

The 2005 senior design team at CSU is proposing the use of a remotely controllable robotic arm to give added mobility and function to an infrared camera. The ability to use the camera as an extension away from the main body of the robot as well as to give it four degrees of freedom in mobility would provide great advantages in the overall function of the robot. However, these advantages are constrained by the need to keep the robot lightweight and durable.



A collapsible linkage design was chosen. It consists of a set of scissor-type linkages driven together by a motor with a threaded shaft. This design has a very advantageous maximum extension. With a swivel mount, the camera would have a wide range of motion. Problems with this design could include failure, and stability.

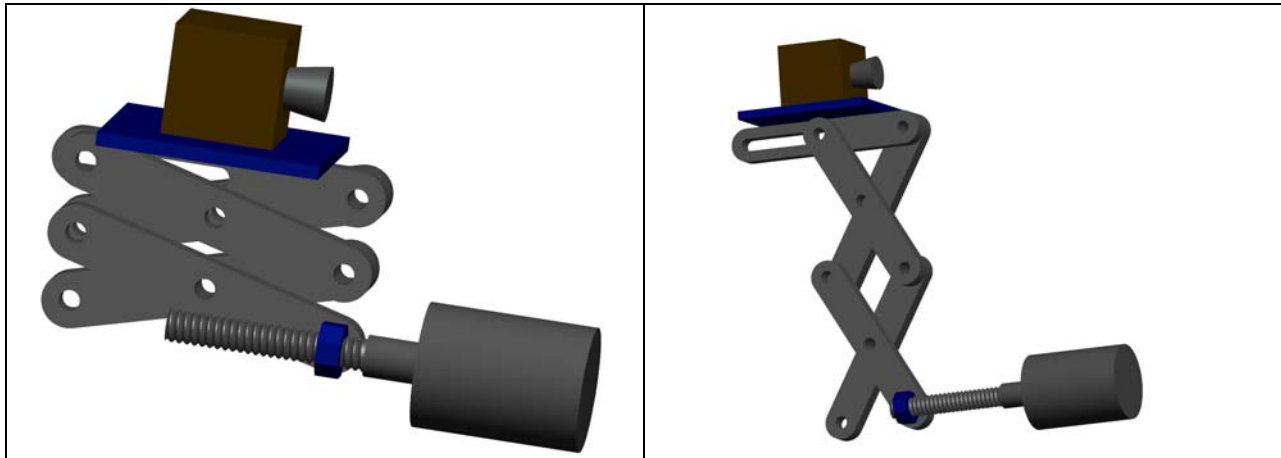


Fig 3. Model Collapsed.

Fig 4. Model Extended.

The design of the linkage could help alleviate these problems. If a design and material combination are found to be effective, it would make the arm sturdy and able to withstand impact from falls. That is the goal of this design experiment.

Problem Statement

Team:

The objective of this project is to develop a mobile robotic arm capable of holding and carrying a portable infrared camera. The arm must be lightweight, reliable, impact resistant and capable of withstanding abnormal temperature conditions. It must have the capacity to be controlled autonomously from the main body and, when not in use, it should be able to fit into the chassis design without interfering with the functions of the rest of the robot.

Individual:

The objective of this individual design report is to design a linkage capable of withstanding a high impact force without damage while maintaining a light weight.

Constraints

Objective

- Must be able to detect and identify on a map, where a victim is positioned to within one cubic meter. (Competition Rule).
- Must be able to setup and break down robot and equipment in less than 10 minutes. (Competition Rule).
- Be able to identify simulated heat signatures of human body temperatures by victims, $95^{\circ} \pm 15^{\circ}$ F. (Competition Rule).
- Be able to detect and identify human equivalents of CO₂ emissions produced by the victims, of approximately ≥ 2000 ppm. (Competition Rule).
- Be able to detect sound from victims in a given decibel range of 100-55 dB.
- Be able to distinguish sounds such as shouting, tapping, moaning, shuffling, and crying of child victims from ambient noise in the arena and caused by the robot itself.
- Power supply must be able to last duration of mission, 20 min.
- Robot must be able to traverse a set of up to 8 stairs, with each step measuring approximately 2.75 ft x 0.7 ft x 0.7 ft.
- Must be less than 35 inches wide at its widest point. (Competition Rule).
- Must be able to turn around inside of a 35 in. wide hallway. (Competition Rule).
- Must not move any structural portion of the arena.
- Must not move environmental elements, such as bricks, ramps, pallets, leaning debris, movable debris or apparently heavy or sharp objects off of, on to, or push into the victims, or push the victims off of platforms.
- All pinch points must be labeled.

Subjective

- Be able to detect and avoid barriers.
- Be able to detect and identify human life signs.
- Be able to determine the state of consciousness of victim. (Competition Rule).
- Be able to determine the situation of victim (level of visibility). (Competition Rule).
- Be able to create quality map with victim locations.
- Must identify pathways to other arenas.
- Be able to operate autonomously when situation is required.

Criteria

Objective-quantitative

- Total arm assembly should weigh maximum of 2 lbs. The total weight of the robot must not exceed 50 lbs.
- The arm and camera must not consume more than 10 W of power.
- Four degrees of freedom must be delivered to the camera including rotational mobility of 360°.
- Maximum length including camera should not exceed 12 inches when compacted and 24 inches when extended. This will allow for an easier integration into the body of the robot.

Subjective-qualitative

- Main body functions and subsystems, such as speed, movement, sensing, and viewing should not be interfered with.
- The arm should not expose the circuitry and motors to harmful elements such as dust, water, debris, etc.
- The arm must be able to withstand impact of body mass dropped from three feet.

Preliminary Design

A preliminary design of a basic linkage with three holes has been chosen to start with. This basic linkage would be used for impact testing and material selection. Ideally, after the material is selected, the link would be modified in multiple ways to find a more optimal design to decrease the weight of the linkage system.

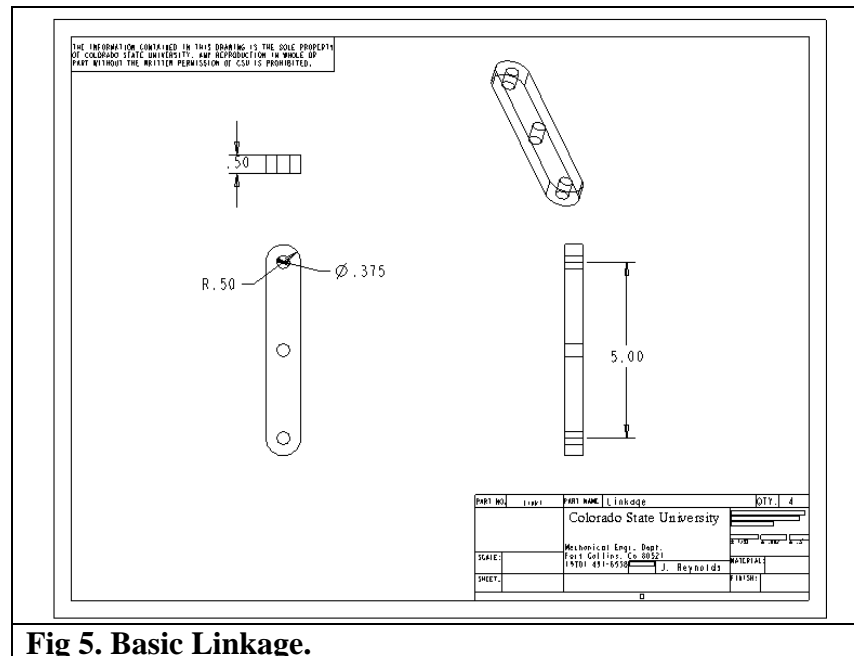


Fig 5. Basic Linkage.

Material Selection

Analysis

The materials considered here will be stainless steel, 6061 aluminum, nylon, reinforced impact grade nylon, and carbon graphite. This will provide a wide range of materials with high strength and/or high flexibility. To ensure that the materials are compared accurately and effectively, Pro/Mechanica will be used for analysis. The robotic arm must be able to withstand a three foot fall. The maximum allowable weight for the robot itself is 50 pounds. Therefore, the full weight of 50 pounds will be considered in the case of the robot falling inverted from three feet high with the entire weight landing on the linkage system. The impact force will be calculated using a combination of Equation 1 and Pro/Mecanica.

$$F_e := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right)$$

Equation 1. Impact Force.[6]

In Equation 1, F_e is the force at impact. W is the weight that would cause a static deflection, δ_{st} , and h is the height of the drop. Because of the three pinned joints and the force acting on an angle during this type of impact, it is not a simple task to use a static or dynamic analysis on this link. Therefore, Pro/Mechanica will be used to calculate the static deflection. That static deflection is then used in the equation to calculate the force at impact. This force is then placed back into the analysis to calculate the maximum Von Mises stress in the link during impact. This three-dimension combination of principle stresses can then be used to compare to the yield strength of the material chosen. This will validate a material's availability for this application.

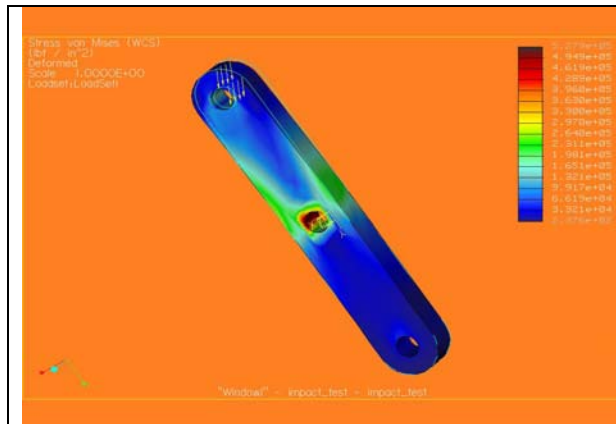


Fig 6. Stainless Steel.

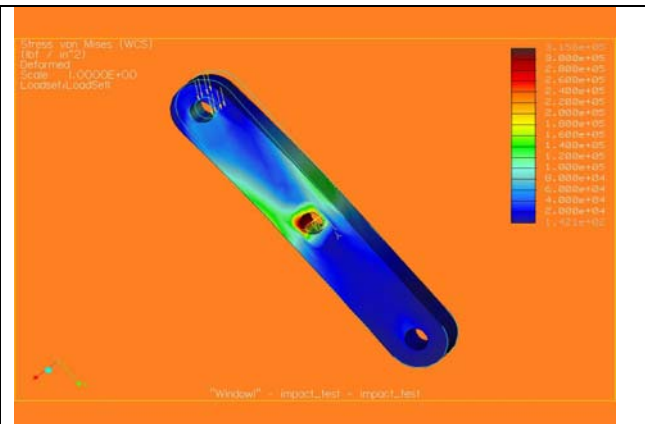


Fig 7. 6061 Aluminum.

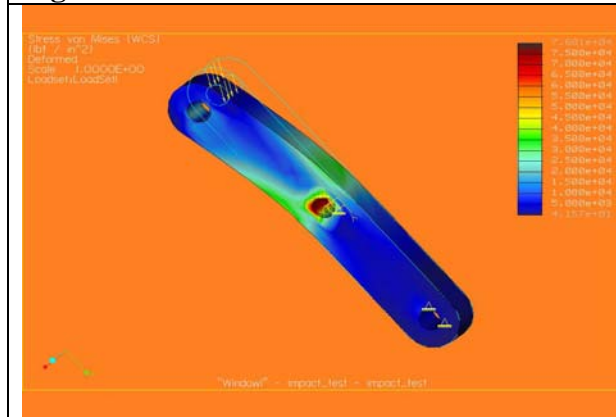


Fig 8. #6 Nylon.

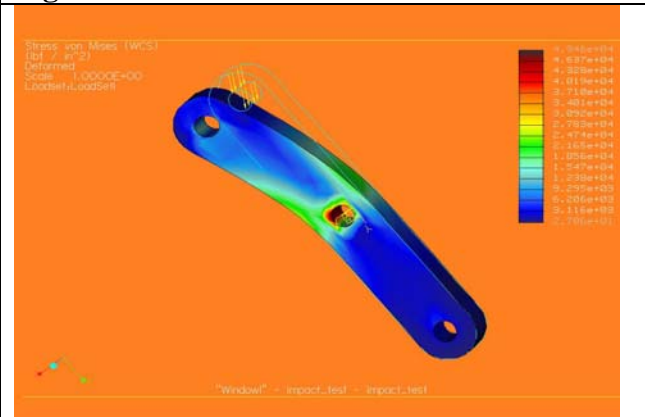


Fig 9. Impact Grade Nylon Reinforced w/ Glass

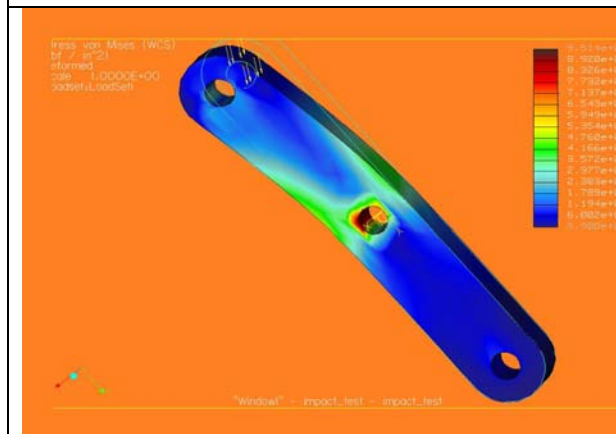


Fig 10. Carbon Fiber.

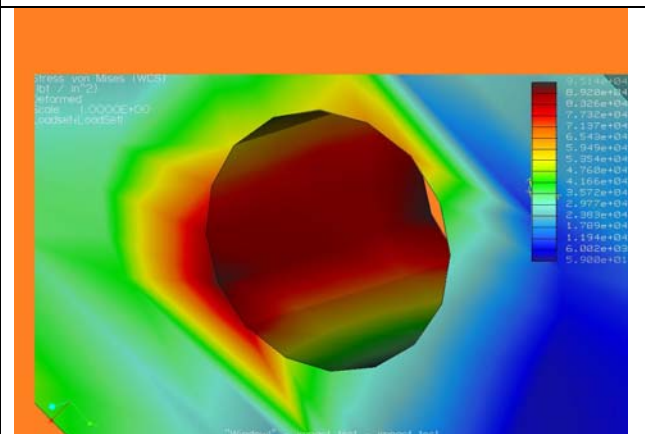


Fig 11. Carbon Fiber, Zoom on Center Hole.

Results

Figures six through ten show the result of the impact on the link as different materials. These fringe plots show the Von Mises stresses as they are distributed throughout the link. These figures also show the link deformed to the maximum deflection that is seen during the impact. This deflection is scaled to 1 to show the actual bending. The calculations necessary for these analyses can be found in Appendix A.

Stainless Steel

The stainless steel bar had the smallest amount of deflection, 0.084 in., out of all the materials tested. However, as can be seen by the inverse relationship in Equation 1 between F_e and δ_{st} , the lower the deflection is, the higher the effective force will be. In fact the effective force ended up being 21,490 lbf. This yielded a maximum Von Mises stress of $5.28 \cdot 10^5$ lbf/in². The yield strength of stainless steel is only $5.58 \cdot 10^4$ lbf/in². [4] This means that part will yield at every point on this link that is not colored dark blue. Even though the yield is very small, it does change the shape of the link and because of the substantial weight of stainless steel, other materials should be considered.

6061 Aluminum

The aluminum link deflected a maximum of 0.141 inches resulting in an impact force of 12,850 lbf. This produced a max stress of $3.16 \cdot 10^5$ lbf/in². This is higher than the aluminum's yield strength of $3.7 \cdot 10^4$ lbf/in² by an order of magnitude. [4] Just like the stainless steel, in the figure above, every part of the link that is not dark blue is yielding. This makes aluminum an even worse material for this application because of its larger deflection

Nylon

Nylon had a much higher deflection of 0.593 inches. This resulted in a much lower impact force of 3,085 lbf which resulted in a max Von Mises stress of $7.6 \cdot 10^4$ lbf/in². The yield strength of $1.02 \cdot 10^4$ lbf/in² is still lower than the max Von Mises stress, but it is much closer than the materials considered thus far. [4] In Figure 8, once again, only the dark blue area of the link will not yield. Because this material seemed to perform better, a higher strength nylon will be considered next.

Impact Grade Nylon Reinforced with Fiber Glass

This "impact grade" nylon has a much higher yield strength of $2.03 \cdot 10^4$ lbf/in² than the nylon previously considered. [4] This nylon also had the highest deflection of all the materials tested at 0.919 inches. This severe deflection can be easily seen in Figure 9. The resulting effective impact force was only 2,008 lbf which yielded a max stress of $4.95 \cdot 10^4$ lbf/in². This is

still higher than the yield strength but only by a factor of two. The green, yellow, and red areas in Figure 8 are the only areas that will yield.

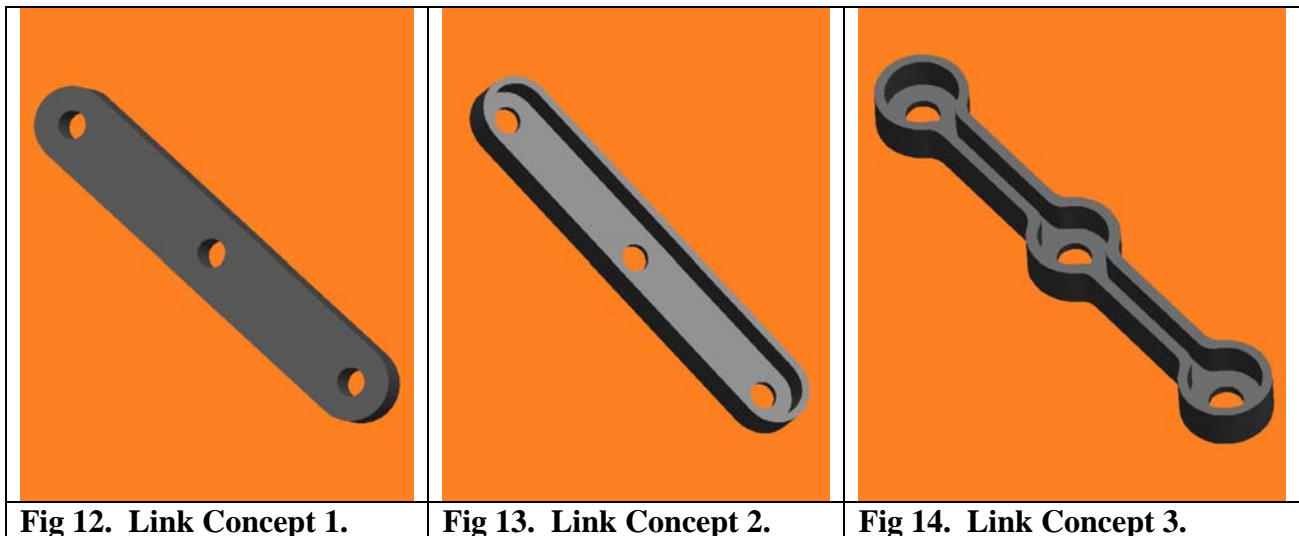
Carbon Fiber

The next material considered was carbon fiber. This is because of its very high yield strength of $8.4 \cdot 10^4$ lbf/in² and low modulus of elasticity.[4] Carbon fiber is not one of Pro/Mechanica's default materials and the Poisson's Ratio was researched to be 0.498.[5] The carbon fiber deflected a maximum of 0.472 inches. This resulted in a calculated impact force of 3,866 lbf. The maximum Von Mises stress was $9.51 \cdot 10^4$ lbf/in². This is slightly higher than the yield strength, but according to Figures 10 & 11 this maximum only appears in the very dark red areas of the fringe plot. This is only at a small part of the inside of the middle hole.

Because the yielding in this link is very small for such a powerful impact, this material will be used. Even though this gives a safety factor of only about 1 for this case, the normal operating conditions of lifting a small camera up and down ends in a safety factor of about 1000. It is also important to realize that if this were to fail, human life is not directly in danger.

Weight Reduction

Three different link designs were considered. The first is simply the basic link. The other two links have material removed from them in an attempt to make them lighter. Each of the designs would then be tested with the impact force of dropping a 50 lb weight on it from three feet high. Then, if the designs appear to be promising, they are analyzed by Pro/Mechanica using an optimization algorithm to adjust certain dimensions until an optimum set of values are found. It is understood that any result would most likely not be the best optimum answer for the problem, but that it would be more optimized than the previous design.



Concept Verification

The following three fringe plots show the max stresses of the three concepts while show the maximum deformation on a scale of 1.

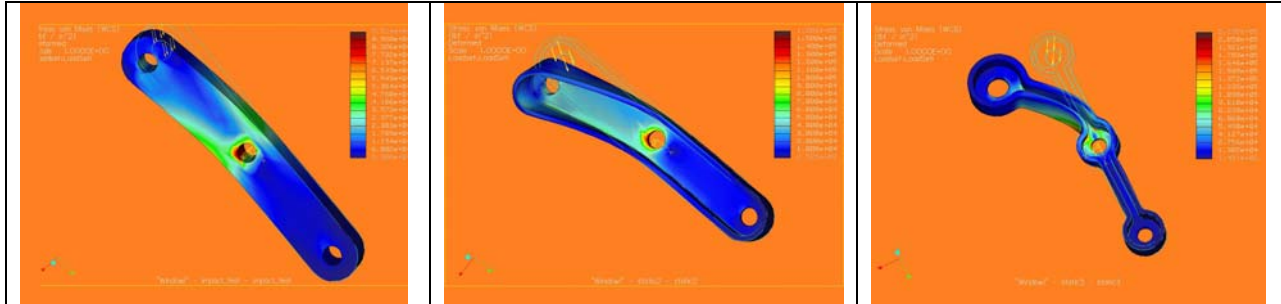


Fig 15. Concept 1 Deformed.

Fig 16. Concept 2 Deformed

Fig 17. Concept 3 Deformed.

Optimization

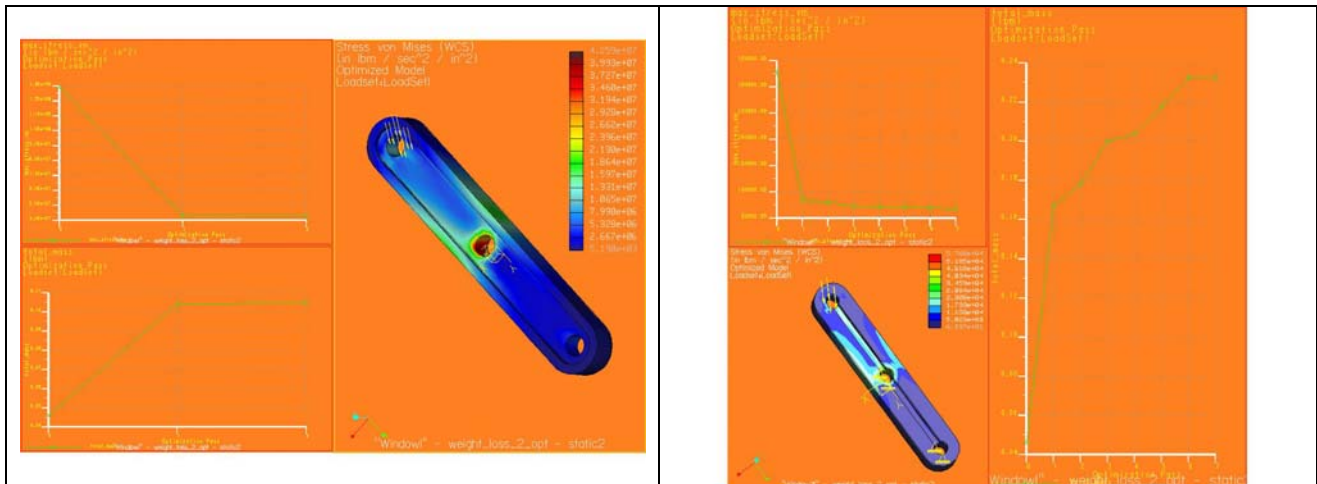


Figure 18. Concept 2 After Optimization.

Figure 19. Concept 3 After Optimization.

For each optimization, the primary goal was to reduce the maximum Von Mises stress. The weight was limited in each case, but the optimizations led each of Concept 1 & 2 more toward the simple link, Concept 1. The Concept 1 optimization simply forced to grow to its maximum limits. This is most likely because the maximum Von Mises stress for each concept was already slightly larger than the prescribed limit of failure, the yield strength for the carbon fiber material. Because of these results, the optimum choice would be to simply use an unmodified version of Concept 1.

Conclusions

Based on the constraints and criteria, a design for a linkage was developed. This design was then used to select a material. Once a material was selected, three different concepts were used for an optimization program. The finite element analysis impact study for the material selection led to the use of a very strong polymer known as carbon fiber. This material performed the better under impact than stainless steel, 6061 aluminum, nylon, and reinforced nylon. The carbon fiber material was used for the 3-D Pro/Engineer models so that an accurate comparison could be made between the three concept designs using Pro/Mechanica. First, the impact analysis was used on these designs; then, they were used in an optimization analysis to see if the weight could be reduced further. These optimization studies led back to the use of the original simple link design.

Using this linkage design for the robotic arm considered should prove to work well. This linkage will allow the robotic arm to lift the camera very effectively without significantly increasing the weight or the risk of failure.

References

1. 2004 RoboCup Rescue Rules and Scoring Metric. 2 March 2005. <<http://robotarenas.nist.gov/rules.htm>>.
2. Kaiser, Carl. Personal Interview. 18 February 2005.
3. 2004 Roach Team. Colorado State University. *Roach Walking Machine*. 7 May 2004.
4. MatWeb Material Property Data. <<http://www.matweb.com>>. <<http://matweb.com/search/SpecificMaterial.asp?bassnum=O1780>>.
5. *Finite Element Modeling of 3-D Braided Carbon Fiber/Urethane Elastomer Tubes*. MatWeb. <<http://fiberarchitects.com/aerospace/braidtubes.html>>.
6. Juvinall, Robert C., and Kurt M. Marshek. Fundamentals of Machine Component Design. Wiley, 2002.

Appendix A – Impact Calculation

Stainless Steel 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad F_x = w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \quad F_y = w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \quad \text{MPa} := 10^6\text{ Pa} \quad h := 3\text{ ft}$$

$$\delta_{st} := 9.763 \cdot 10^{-5}\text{ in} \quad F_e := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad F_e = 2.149 \times 10^4\text{ lbf}$$

$$F_{e_x} := F_e \cdot \sin(30\text{deg}) \quad F_{e_y} := F_e \cdot \cos(30\text{deg}) \quad F_{e_x} = 1.075 \times 10^4\text{ lbf} \quad F_{e_y} = 1.861 \times 10^4\text{ lbf}$$

$$\delta_{max} := \delta_{st} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad \delta_{max} = 0.084\text{ in} \quad S_{y_{max}} := 385\text{ MPa}$$

$$S_{y_{max}} = 5.584 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{max}} := 5.28 \cdot 10^5 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{max}} - S_{y_{max}} = 4.722 \times 10^5 \frac{\text{lbf}}{\text{in}^2}$$

Aluminum 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad F_x = w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \quad F_y = w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \quad h := 3\text{ ft}$$

$$\delta_{st} := 2.734 \cdot 10^{-4}\text{ in} \quad F_e := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad F_e = 1.285 \times 10^4\text{ lbf}$$

$$F_{e_x} := F_e \cdot \sin(30\text{deg}) \quad F_{e_y} := F_e \cdot \cos(30\text{deg}) \quad F_{e_x} = 6.427 \times 10^3\text{ lbf} \quad F_{e_y} = 1.113 \times 10^4\text{ lbf}$$

$$\delta_{max} := \delta_{st} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad \delta_{max} = 0.141\text{ in} \quad S_{y_{max}} := 255\text{ MPa}$$

$$S_{y_{max}} = 3.698 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{max}} := 3.156 \cdot 10^5 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{max}} - S_{y_{max}} = 2.786 \times 10^5 \frac{\text{lbf}}{\text{in}^2}$$

Nylon 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad F_x \quad F_y$$

$$w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \quad w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \quad h := 3\text{ ft}$$

$$\delta_{st} := 4.805 \cdot 10^{-3}\text{ in} \quad Fe := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad Fe = 3.085 \times 10^3\text{ lbf}$$

$$Fe_x := Fe \cdot \sin(30\text{deg}) \quad Fe_y := Fe \cdot \cos(30\text{deg}) \quad Fe_x = 1.543 \times 10^3\text{ lbf} \quad Fe_y = 2.672 \times 10^3\text{ lbf}$$

$$\delta_{max} := \delta_{st} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad \delta_{max} = 0.593\text{ in} \quad Sy_{max} := 70\text{MPa}$$

$$Sy_{max} = 1.015 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} := 7.601 \cdot 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} - Sy_{max} = 6.586 \times 10^4 \frac{\text{lbf}}{\text{in}^2}$$

Nylon Glass Filled Impact Grade 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad F_x \quad F_y$$

$$w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \quad w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \quad h := 3\text{ ft}$$

$$\delta_{st} := 1.144 \cdot 10^{-2}\text{ in} \quad Fe := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad Fe = 2.008 \times 10^3\text{ lbf}$$

$$Fe_x := Fe \cdot \sin(30\text{deg}) \quad Fe_y := Fe \cdot \cos(30\text{deg}) \quad Fe_x = 1.004 \times 10^3\text{ lbf} \quad Fe_y = 1.739 \times 10^3\text{ lbf}$$

$$\delta_{max} := \delta_{st} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad \delta_{max} = 0.919\text{ in} \quad Sy_{max} := 140\text{MPa}$$

$$Sy_{max} = 2.031 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} := 4.946 \cdot 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} - Sy_{max} = 2.915 \times 10^4 \frac{\text{lbf}}{\text{in}^2}$$

Carbon Fiber 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad F_x \quad F_y$$

$$w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \quad w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \quad h := 3\text{ ft}$$

$$\delta_{st} := 3.051 \cdot 10^{-3}\text{ in} \quad Fe := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad Fe = 3.866 \times 10^3\text{ lbf}$$

$$Fe_x := Fe \cdot \sin(30\text{deg}) \quad Fe_y := Fe \cdot \cos(30\text{deg}) \quad Fe_x = 1.933 \times 10^3\text{ lbf} \quad Fe_y = 3.348 \times 10^3\text{ lbf}$$

$$\delta_{max} := \delta_{st} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{st}}} \right) \quad \delta_{max} = 0.472\text{ in} \quad Sy_{max} := 580\text{MPa}$$

$$Sy_{max} = 8.412 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} := 9.514 \cdot 10^4 \frac{\text{lbf}}{\text{in}^2} \quad vm_{max} - Sy_{max} = 1.102 \times 10^4 \frac{\text{lbf}}{\text{in}^2}$$

Concept Design Calculations

Carbon Fiber 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad \begin{matrix} \text{Fx} \\ w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \end{matrix} \quad \begin{matrix} \text{Fy} \\ w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \end{matrix} \quad h := 3\text{ ft}$$

$$\delta_{\text{st}} := 5.18 \cdot 10^{-2} \text{ in} \quad F_e := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{\text{st}}}} \right) \quad F_e = 957.39\text{ lbf}$$

$$F_{e_x} := F_e \cdot \sin(30\text{deg}) \quad F_{e_y} := F_e \cdot \cos(30\text{deg}) \quad F_{e_x} = 478.695\text{ lbf} \quad F_{e_y} = 829.124\text{ lbf}$$

$$\delta_{\text{max}} := \delta_{\text{st}} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{\text{st}}}} \right) \quad \delta_{\text{max}} = 1.984 \text{ in} \quad S_{y_{\text{max}}} := 580\text{ MPa}$$

$$S_{y_{\text{max}}} = 8.412 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{\text{max}}} := 9.514 \cdot 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{\text{max}}} - S_{y_{\text{max}}} = 1.102 \times 10^4 \frac{\text{lbf}}{\text{in}^2}$$

Carbon Fiber 30 degrees from vertical

$$w := \frac{50\text{ lbf}}{2} \quad \begin{matrix} \text{Fx} \\ w \cdot \sin(30\text{deg}) = 12.5\text{ lbf} \end{matrix} \quad \begin{matrix} \text{Fy} \\ w \cdot \cos(30\text{deg}) = 21.65\text{ lbf} \end{matrix} \quad h := 3\text{ ft}$$

$$\delta_{\text{st}} := 1.317 \cdot 10^{-2} \text{ in} \quad F_e := w \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{\text{st}}}} \right) \quad F_e = 1.874 \times 10^3 \text{ lbf}$$

$$F_{e_x} := F_e \cdot \sin(30\text{deg}) \quad F_{e_y} := F_e \cdot \cos(30\text{deg}) \quad F_{e_x} = 936.822\text{ lbf} \quad F_{e_y} = 1.623 \times 10^3 \text{ lbf}$$

$$\delta_{\text{max}} := \delta_{\text{st}} \cdot \left(1 + \sqrt{1 + \frac{2 \cdot h}{\delta_{\text{st}}}} \right) \quad \delta_{\text{max}} = 0.987 \text{ in} \quad S_{y_{\text{max}}} := 580\text{ MPa}$$

$$S_{y_{\text{max}}} = 8.412 \times 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{\text{max}}} := 9.514 \cdot 10^4 \frac{\text{lbf}}{\text{in}^2} \quad v_{m_{\text{max}}} - S_{y_{\text{max}}} = 1.102 \times 10^4 \frac{\text{lbf}}{\text{in}^2}$$

